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Environment-mediated quantum state transfer

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Abstract

We propose a scheme for quantum state transfer (QST) between two qubits, which is based on their individual interaction with a common boson environment. The corresponding single-mode spin-boson Hamiltonian is solved by mapping it into a wave propagation problem in a semi-infinite ladder and the fidelity is obtained. High fidelity occurs when the qubits are equally coupled to the boson while the fidelity becomes smaller for nonsymmetric couplings. The complete phase diagram for such an arbitrary QST mediated by bosons is discussed.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

One of the most interesting problems in the area of quantum information is how to transfer a quantum state from one location to another. For example, QST from A to B can be done via quantum teleportation [1] only if a prior connection between the remote places is established by letting A and B have one from two auxiliary entangled qubits. A more direct method should involve flying qubits, usually photons in optical fibers, which can send directly quantum information between two distant locations of a quantum computer [2]. However, the latter approach would be very difficult to realize experimentally since it requires an interface between the optical system and the hardware where a quantum computation takes place. A very successful QST was pioneered in [3] which includes the locations A and B and also the quantum transfer ground into the same system. In this case the quantum information of the qubits at one end of the chain propagate via the interaction between the components of a permanently coupled physical system or a quantum graph. A perfect or nearly perfect QST occurs between two local spins in quantum spin chains and networks, at least for short distances [3–6]. The protocol for such a quantum communication relies on the fact that the auxiliary device which plays the role of the quantum channel or quantum bus is the medium itself. For example, the exchange interactions of a quantum spin chain can allow transfer of quantum information between qubits which belong to the first and last local spins while the rest of the chain acts as the quantum channel. In this case the exchange of information is

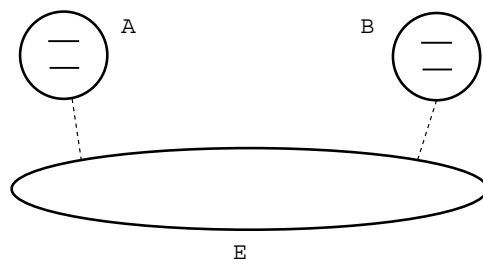


Figure 1. The proposed protocol for a QST between two qubits A and B represented by two-level systems which interact with a common bosonic environment E which acts as the quantum transfer channel.

achieved via magnon elementary excitations. Physical devices for such QST could be built, at least in principle, and since no external control is required they can overcome possible decoherence mechanisms.

In this paper we show that a boson environment could be used to transfer efficiently a quantum state by acting as a quantum channel. It is known [7] that entanglement can be introduced between two qubits if both are independently coupled to a common heat bath with many degrees of freedom. We shall show that even the simplest possible boson environment which consists of one mode can also provide an efficient QST mechanism. For this purpose a spin-boson Hamiltonian is introduced [8, 9] known for many applications in physics and chemistry. A closely related model involving the exchange of quantum information and entanglement between two harmonic oscillators through a bosonic or spin-ring data bus was proposed in [10, 11]. A spin-boson model allowed Caldeira and Leggett [12] to study decoherence via dissipation through a weak coupling of the spin to many bosons, representing a universal realization of a physical environment. In the case of weak spin-boson interaction the excitations within the boson heat bath could be ignored and the problem can be solved leading to decoherence. Our spin-boson model can be regarded as an extension of [7] where two qubits coupled to a common heat bath become entangled with each other. We show that despite the absence of a direct interaction between them their coupling to a simple boson environment mediates an efficient QST. Environment mediated quantum control for a related multi-mode system has been performed in [13].

The proposed spin-boson model allows high fidelity QST between two distant locations by choosing suitable parameters. In order to make the problem tractable we choose the simplest possible quantum channel which consists of a single-mode boson environment. This is the first approximation to a full multi-mode Hamiltonian considered in [8] by replacing the coupling to many modes by coupling to a single effective boson. Our study proceeds as follows: in section 2 the proposed spin-boson model is introduced with a double two-level system Hamiltonian coupled to a single boson. In section 3 a formula is derived for the fidelity of a QST which is obtained by mapping the system onto a wave-propagation problem in a semi-infinite ladder. The results of our calculations with the display of the corresponding phase-diagram and a discussion about the efficiency of the scheme follow in section 4. Finally, in section 5 we discuss possible improvements, extensions and applications.

2. Model and average fidelity

The studied system is illustrated in figure 1. The qubits A and B are not directly coupled with each other but are connected via an auxiliary boson environment E both having nonzero

interaction with E . The qubits in A and B can be represented by two local spins and E acts as the quantum channel. Of course, if E is replaced by a quantum spin chain the model reduces to the one studied in [3]. The Hamiltonian ($\hbar = 1$) is given by the sum

$$H = \omega_A^0 \sigma_A^z + \omega_B^0 \sigma_B^z + \omega b^\dagger b + \lambda_A (b + b^\dagger) \sigma_A^x + \lambda_B (b + b^\dagger) \sigma_B^x,$$

with the qubits in A and B modeled by two-level systems of separations ω_A^0, ω_B^0 , the quantum channel described by a single-boson mode environment of frequency ω and nonzero linear couplings λ_A and λ_B exist between the qubits and the boson channel E , with $\sigma^{x/y/z}$ the corresponding Pauli matrices. Note the similarity of H to a multi-mode model used to study entanglement between the qubits in quantum control theory [13]. The main differences between the present study and [7, 13] lie in the number of modes and the presence or absence of couplings between the qubits and the quantum channel. We consider the nonzero spin-boson couplings λ_A and λ_B since they are expected to be comparable to the two-level separations ω_A^0 and ω_B^0 .

The single-mode Hamiltonian H , although simple enough, cannot be solved exactly. The Hilbert space consists of a direct product of three parts with basis states $|\eta_A, \eta_B, m\rangle$, where $\eta_{A/B} = 0, 1$ label the qubits and $m = 0, 1, 2, 3, \dots$ is the single phonon excitation number of the states in the quantum channel. The QST in this system can be studied similarly to that in a spin network [3]. Suppose that at time $t = 0$ an unknown state $|\psi_A\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$ with parameters θ, ϕ , is generated at qubit A and has to be transferred to B . We also initialize the state of the qubit B to $|0\rangle$ and the state of the quantum channel E to its lowest boson state $|0\rangle$. The initial state of the whole system is $|\psi_A, 0, 0\rangle$ which is separable. When evolution takes place the final state at time t in general becomes a non-separable mixed state. The measurement of the state of qubit B is described by its *reduced density matrix* and both the efficiency and the quality of quantum communication can be obtained by evaluating the corresponding *fidelity* [3].

The fidelity is usually computed by taking average over all pure input states $|\psi_A\rangle$ in their corresponding Bloch sphere

$$\mathcal{F}(t) = \frac{1}{4\pi} \int d\Omega \langle \psi_A | \rho_B(t) | \psi_A \rangle,$$

where the state of A to be transferred is $|\psi_A\rangle$, $\rho_B(t)$ is the reduced density matrix of the qubit B at time t and the average is over all initial $|\psi_A\rangle$. If we let the system evolve for a period of time T , one can find the maximum average fidelity \mathcal{F}_m from the time taken for the average fidelity to reach its first peak corresponding to the maximum fidelity. The peak time t_p is the second important quantity which can characterize a quantum channel, the first being the average fidelity \mathcal{F}_m . High fidelity implies better quantum channel for QST while shorter time to reach the peak means faster QST. If \mathcal{F}_m becomes exactly unity we have perfect QST¹ with the quantum state transferred from A to B without any loss of quantum information.

The reduced density matrix for qubit B can be written as

$$\rho_B = \text{Tr}_{A,E}[\rho_t]$$

by tracing out A and E of the evolved total density matrix $\rho_t = U(t, 0)\rho_0 U(0, t)$, with initial value $\rho_0 = \rho_A \otimes \rho_B \otimes \rho_E$ and time evolution operator $U(t, 0) = e^{iHt}$. This allows us to calculate the average fidelity for any time t , which from now on we shall simply call fidelity. As it stands this formula is rather complicated to perform an analytic evaluation. In the next section the problem is mapped onto an equivalent wave propagation involving two ladders and the corresponding fidelity is written as a function of waves propagating in these ladders.

¹ An average fidelity equal to unity means the final state is different from the initial state to be transferred by a phase factor. Since this phase is known via a simple rotation the final state becomes exactly the same as the initial state.

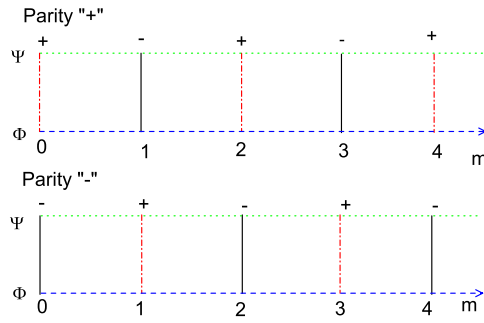


Figure 2. The equivalent problem of wave propagation in ‘+’ and ‘-’ ladders. The nodes denote the basis states and the lines the hoppings with the green lines $\sqrt{m}(\lambda_A + \lambda_B)$, the blue lines $\sqrt{m}(\lambda_A - \lambda_B)$, the red lines $\omega_B^0 + \omega_A^0$ and the black lines $\omega_B^0 - \omega_A^0$ with on-site energies ωm .

3. Wave propagation

A parity symmetry present in H simplifies the Hamiltonian [9] making it block-diagonal in a suitable two-qubit Bell states basis

$$|\Psi_{\pm}, m\rangle = \frac{1}{\sqrt{2}}(|00m\rangle \pm |11m\rangle) \quad |\Phi_{\pm}, m\rangle = \frac{1}{\sqrt{2}}(|01m\rangle \pm |10m\rangle),$$

where $|0\rangle$ and $|1\rangle$ stand for the eigenstates of σ_x . The Hamiltonian which acts on this basis gives

$$\begin{aligned} H|\Psi_{\pm}, m\rangle &= \omega m|\Psi_{\pm}, m\rangle + (\omega_B^0 \pm \omega_A^0)|\Phi_{\pm}, m\rangle + \sqrt{m}(\lambda_A + \lambda_B)|\Psi_{\mp}, m-1\rangle \\ &\quad + \sqrt{m+1}(\lambda_A + \lambda_B)|\Psi_{\mp}, m+1\rangle \\ H|\Phi_{\pm}, m\rangle &= \omega m|\Phi_{\pm}, m\rangle + (\omega_B^0 \pm \omega_A^0)|\Psi_{\pm}, m\rangle + \sqrt{m}(\lambda_A - \lambda_B)|\Phi_{\mp}, m-1\rangle \\ &\quad + \sqrt{m+1}(\lambda_A - \lambda_B)|\Phi_{\mp}, m+1\rangle. \end{aligned}$$

The states split into two, having zero matrix elements between each other and the block-diagonal Hamiltonian matrix is illustrated via two decoupled ladders in figure 2. The states are represented by nodes and the hoppings between nodes by the connecting lines. Note that the ladders in figure 2 are rather similar to each other, the only difference being the ordering of red and black lines. This was very helpful for our calculation (given in the appendix) since on this basis the computations simplify.

The obtained formula for the fidelity can be written in the form

$$\mathcal{F} = \frac{1}{24} \sum_m (\text{Tr}[A_m^\dagger A_m] + \text{Tr}[B_m^\dagger B_m] + \text{Tr}[C_m^\dagger C_m])$$

with

$$\begin{aligned} A_m &= f_m(+, t) + \sigma^z f_m(-, t) & B_m &= f_m(+, t) + (-i\sigma^y) f_m(-, t) \sigma^x \\ C_m &= \sigma^{z1} f_m(+, t) + \frac{\sigma^+}{2} f_m(+, t) \sigma^x + \frac{\sigma^-}{2} f_m(-, t) + \sigma^{z2} f_m(-, t) \sigma^x \\ \sigma^{z1} &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \sigma^{z2} &= \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned}$$

where $\sigma^{+, -, z}$ are the Pauli matrices and $f_m(\pm, t)$ is the propagator in the ladders shown. In the notation used, e.g., $f_3(+, 2)$ means the propagator from $m = 0$ to slice $m = 3$ at time $t = 2$

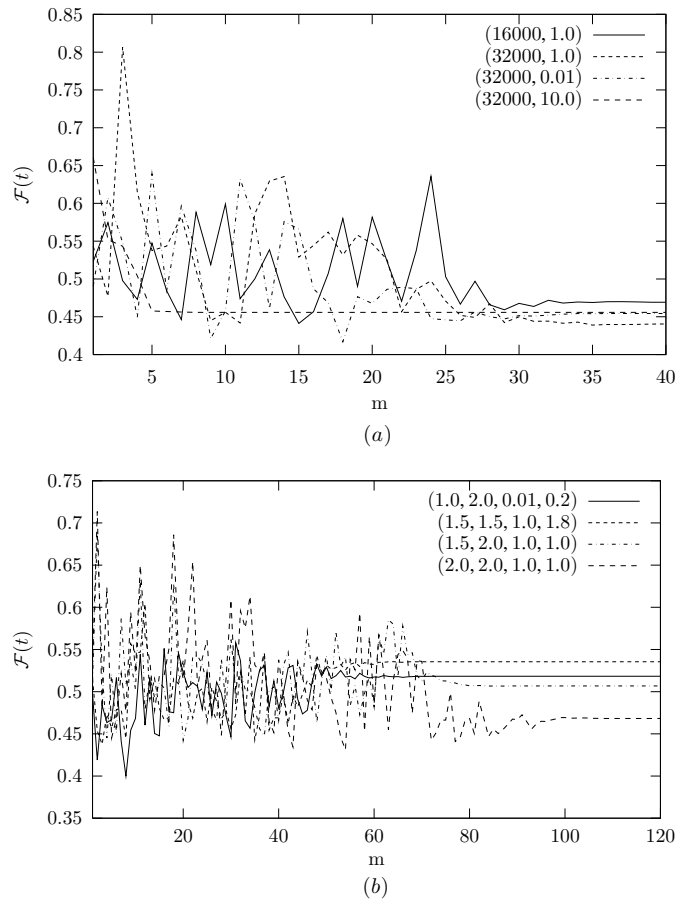


Figure 3. The convergence of the average fidelity $\mathcal{F}(t)$ as a function of the maximum phonon number m : (a) symmetric case with $\lambda_A = \lambda_B = 1.0$ and $\omega_A = \omega_B = \omega_S$ where the parameters in parentheses on the right of the figure are (t, ω_S) , and (b) non-symmetric case where the parameters displayed in parentheses are $(\lambda_A, \lambda_B, \omega_A, \omega_B)$. The maximum time is set at $T = 32000$.

in the ladder with parity '+'. This gives the fidelity of QST written as a linear combination of the propagators in each of the two ladders.

Since both ladders are semi-infinite the corresponding Hilbert spaces must be truncated at a maximum phonon number m . In order to approximate propagation for a very long time long ladders with large maximum m are required. However, a careful study of the formula shows that the fidelity simply arises from the *difference* between the propagators in the ladders. For example, for $\omega_A = 0$ or $\omega_B = 0$ the two ladders are the same and the fidelity becomes precisely zero. Since their structure is rather similar, except for the ordering of lines, if a wave reaches very far from the origin in one of them a very small difference between the two propagators is expected with no contribution to the fidelity. Therefore, accurate computations of the fidelity do not require very long ladders and reasonable maximum m suffices, as it can be seen from figure 3.

The accuracy of the computed results is shown in figure 3(a) by plotting the fidelity as a function of the maximum phonon number m for the symmetric case with $\lambda_A = \lambda_B =$

$\lambda_S, \omega_A = \omega_B = \omega_S$ and in figure 3(b) for the non-symmetric case. The fidelity is shown to converge very rapidly for the maximum phonon numbers $m = 40$ or 50 which permits us to use reasonable coupling strengths. The convergence does not depend on time T and is rather insensitive to $\omega_{A/B}$ depending mostly on the couplings $\lambda_{A/B}$. For example, the numerical results for $\lambda_S = 1.0$ and $\lambda_S = 2.0$ required only $m = 40$ to 50 and more than $m = 100$, respectively. In our computations suitable maximum m was chosen according to the values of $\lambda_{A/B}$ and the convergence was checked by varying m . For couplings λ up to 2 and frequencies ω up to 80 a maximum phonon number m between 50 to 110 turned out to be sufficient.

4. The phase diagram

The quality of QST is determined by the maximum of the average fidelity \mathcal{F}_m in the time period from 0 to T and the time for occurrence of the first peak t_p when the system reaches its maximum. Higher fidelity means more faithful state transfer while the shorter peak time t_p implies faster state transfer. The parameters $\omega_A, \omega_B, \lambda_A$ and λ_B are all in units of $\omega = 1$, while the maximum fidelity and the first peak time are obtained in the time interval $[0, T = 32 \times 10^3]$.

4.1. Symmetric couplings

The phase diagrams of the maximum fidelity and the first peak time are shown in figures 4(a) and (b) as a function of two parameters $\omega_S = \omega_A = \omega_B$ and $\lambda_S = \lambda_A = \lambda_B$. They are divided into the following three regions:

Region I: a weak coupling region which lies in the upper left corner of figure 4(a) where $\omega_S \gg \lambda_S$. In this case the corresponding first peak time t_p shown in figure 4(b) is large equal to the upper bound of the chosen time interval T . In other words, the fidelity never reaches its maximum within the adopted evolution time. This indicates that probably a higher fidelity might occur for even longer times so that we can call this a ‘slow region’. We may conclude that a good state transfer is impossible in this region because of the long time t_p needed.

Region II: lies in the lower part of the figure, which is too small to be seen in figure 4(a) and this plot is magnified in figure 4(c). In this region ω_S and λ_S are of the same order of magnitude so that the fidelity is again low but for a different reason than that of region I. The first peak time in this case from figure 4(b) is less than T and the QST is affected by increasing ω_S . For zero ω_S no QST is possible while it becomes better when increasing the qubit-environment coupling λ_S .

Region III: the rest of figure 4(a). One can see that in the majority of this region high fidelity occurs with the first peak time mainly less than 5×10^3 . This region corresponds to a two-valley Hamiltonian and the system behaves as a good quantum channel.

4.2. Non-symmetric couplings

We have also considered the non-symmetric case where the two couplings and the two frequencies are not equal. The influence of a deviation from equal couplings is studied by choosing $\lambda_S = 0.8, \omega_S = 20.0$ with the corresponding point of the symmetric phase diagram belonging to region III having very high fidelity equal to 0.998. A small deviation $\delta\omega$ in ω_B with $\omega_B = \omega_A + \delta\omega$ is shown in figure 5(a) to influence dramatically the QST, which is extremely sensitive even for deviations of the order of 10^{-4} . The asymmetry in the coupling constants in figure 5(b) has a much smaller effect.

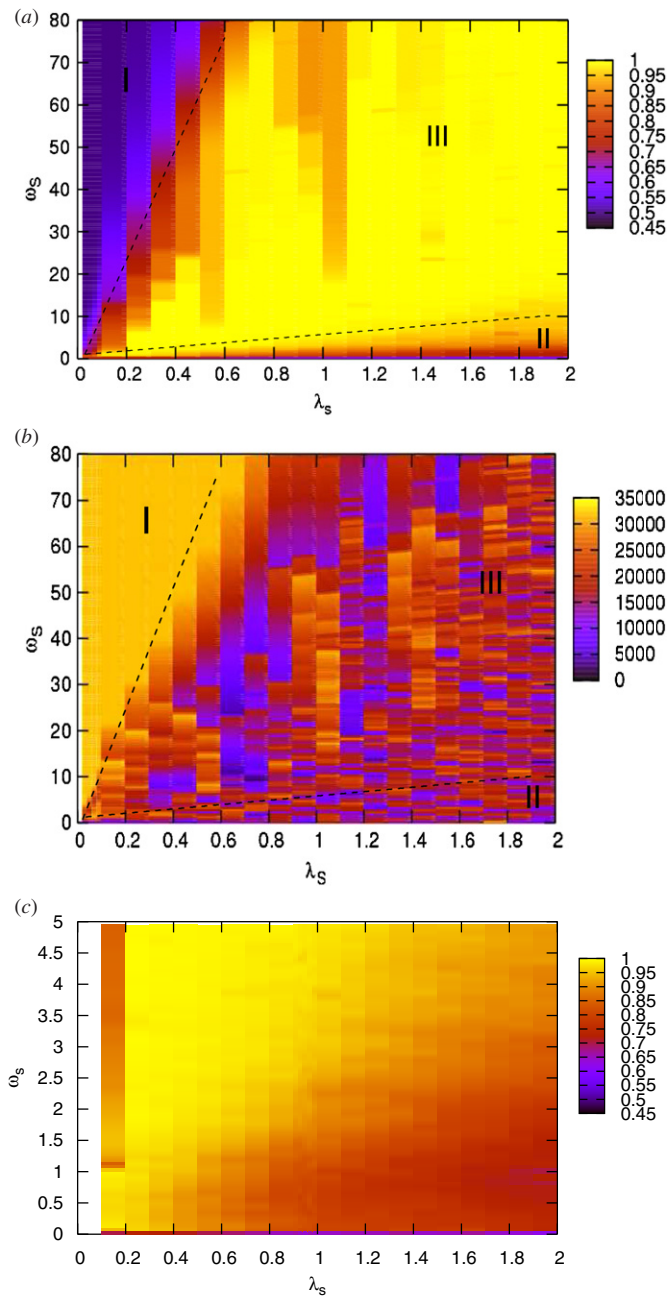


Figure 4. Phase diagram for the fidelity of QST. (a) The phase diagram of the maximum fidelity as a function of equal couplings λ_S and equal qubit separations ω_S . Three regions can be distinguished as explained in the text. (b) The first peak time of the QST as a function of the equal couplings λ_S and the qubit separations ω_S . This picture also has the three regions mentioned in the text. (c) The region II of figure 4(a) is shown in some more detail.

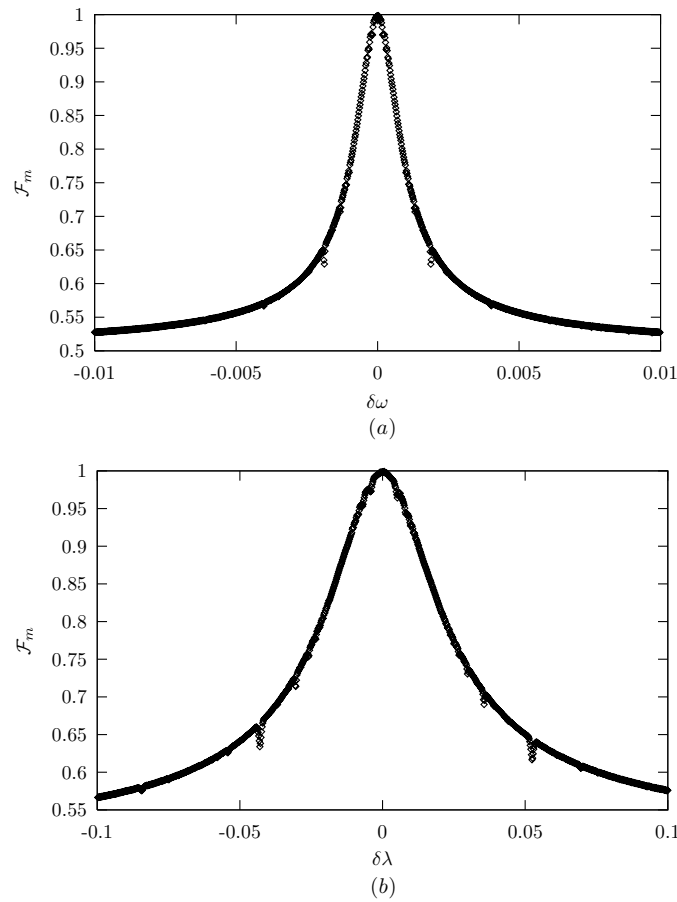


Figure 5. The maximum of the average fidelity $\mathcal{F}_m = 0.998$, a value close to a perfect QST, becomes lower for deviations from equal frequencies $\omega_A = \omega_B = 20.0$ and equal couplings $\lambda_A = \lambda_B = 0.8$, with $\delta\omega = \omega_B - \omega_A$ and $\delta\lambda = \lambda_B - \lambda_A$, respectively. The \mathcal{F}_m is obtained in the region $[0, T = 33 \times 10^3]$.

The sensitivity of the obtained fidelity on the symmetry of the two qubits can be understood as a consequence of the total energy conservation. Since the essential condition for a perfect QST is a separable final state the symmetric configuration is more favorable because in that case the energy, which is equal to its initial value, leads to a factorized final state simply by swapping the initial quantum state between qubits A and B . During the evolution the initial total energy $E = \langle \psi | H | \psi \rangle = \omega_A \cos(\theta)$ becomes $E = \omega_B \cos(\theta) + \dots$ making it obvious why the asymmetric case $\omega_B \gg \omega_A$ does not favor perfect QST. Clearly, any deviation from the symmetric case via mismatch of the two qubit frequency separations or their couplings to the boson decreases the corresponding fidelity, making more difficult the QST.

5. Discussion–conclusions

Although the role of an environment for a quantum system is usually that of causing decoherence the entanglement between the system and the environment also signals the

possibility of quantum information transfer via the environment. We suggest a QST between two qubits via a coupling to a common bosonic medium which acts as the quantum channel. We have derived a formula for the fidelity of the corresponding QST by mapping this into a wave propagation problem, which is much easier to understand and solve. For symmetric couplings or equal frequency separations, high fidelity QST between the two qubits is obtained for a wide range of parameters. We show that small deviations from this symmetry can dramatically lower the QST. This result can be easily understood as a consequence of the total energy conservation during the evolution.

The obtained fidelity for the suggested QST protocol can be further improved by optimizing the proposed settings such as encoding the initial state. This can be done as proposed in [14, 15] by exploiting time-dependent couplings, dual-rail encodings or other similar schemes. Our questions for further study are: (i) possible extensions of the present scheme to include a multimode boson environment since our results can cover only approximately the multimode case, (ii) connections of QST to wave propagation in media also in the presence of disorder which can also give ballistic, chaotic and even localized states (in the latter case QST is impossible) and (iii) possible realization of an experiment where QST mediated by bosons can occur, for example, between two quantum dots coupled to an appropriate phonon environment of a nanostructure.

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Appendix. Derivation of the formula for the fidelity

The average fidelity

$$\mathcal{F}(t) = \frac{1}{4\pi} \int d\Omega \langle \psi_A | \rho_B(t) | \psi_A \rangle$$

over $|\psi_A\rangle$ becomes

$$|\psi_A\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle$$

$$\frac{1}{4\pi} \int d\Omega \dots = \frac{1}{4\pi} \int_0^\pi d\sin(\theta) d\theta \int_0^{2\pi} d\phi \dots$$

The reduced density matrix $\rho_B(t)$ can be calculated via

$$\rho_B = \text{Tr}_{A,E}[\rho_t]$$

$$\rho_t = U(t, 0)\rho_0 U(0, t)$$

$$U(t, 0) = e^{iHt}$$

where, the partial trace over the degrees of freedom for qubit A and the quantum channel E is taken. H is the Hamiltonian for the system $A \otimes B \otimes E$ and $U(t, 0)$ is the corresponding time evolution operator.

To simplify the formula first we have calculated the integral. It is convenient for us to choose *coherent vector representation* [16] to express the density matrix,

$$\rho_B(t) = \frac{1}{2}(1 + \vec{p}_B(t) \cdot \vec{\sigma}) \quad \rho_A = \frac{1}{2}(1 + \vec{p}_A \cdot \vec{\sigma})$$

and assuming the relation between the two coherent vectors

$$\vec{p}_B(t) = T(t) \cdot \vec{p}_A + \vec{T}_0(t)$$

we can carry out the integral

$$\mathcal{F}(t) = \frac{1}{2} \left[1 + \frac{1}{3} \text{Tr}(T(t)) \right].$$

We need to calculate the matrix $T(t)$, e.g., to express the final state of qubit B as a function of initial state of qubit A ,

$$\rho_B(t) = \text{Tr}_{A,E}[U(t, 0)\rho(0)U(0, t)]$$

where $\rho(0)$ is the initial state of the whole system ($A \otimes B \otimes E$). It is separable so that

$$\rho(0) = \rho_A(0) \otimes \rho_B(0) \otimes \rho_E(0)$$

$$\rho_B(0) = |0\rangle\langle 0|$$

$$\rho_E(0) = |0\rangle\langle 0|.$$

By inserting $|\eta_A, \eta_B, m\rangle$ into these formulae we find

$$\rho_B(\eta_B, \eta'_B, t) = \sum_{\bar{\eta}_A, \bar{\eta}'_A} J_B(\eta_B, \eta'_B, \bar{\eta}_A, \bar{\eta}'_A, t) \rho_A(\bar{\eta}_A, \bar{\eta}'_A, 0)$$

$$J_B(\eta_B, \eta'_B, \bar{\eta}_A, \bar{\eta}'_A, t) = \sum_{\eta_A} J(\eta_A \eta_B, \eta_A \eta'_B, t; \bar{\eta}_A 0, \bar{\eta}'_A 0, 0)$$

$$J(\eta_A \eta_B, \eta'_A \eta'_B, t; \bar{\eta}_A \bar{\eta}_B, \bar{\eta}'_A \bar{\eta}'_B, 0) = \sum_m \langle \bar{\eta}_A \bar{\eta}_B, 0 | U(0, t) | \eta_A \eta_B, 0 \rangle \\ \times \langle \eta'_A \eta'_B, 0 | U(t, 0) | \bar{\eta}'_A \bar{\eta}'_B, 0 \rangle,$$

where $\eta_{A/B} = 0/1, m = 0, 1, 2, 3, \dots$

The matrix element between $\rho_B(t)$ and $\rho_A(0)$ is related by the function J_B

$$T = \begin{pmatrix} T^x(01) + T^x(10) & i[T^x(10) - T^x(01)] & T^x(00) - T^x(11) \\ T^y(01) + T^y(10) & i[T^y(10) - T^y(01)] & T^y(00) - T^y(11) \\ T^z(01) + T^z(10) & i[T^z(10) - T^z(01)] & T^z(00) - T^z(11) \end{pmatrix}$$

where,

$$T^x(\eta\eta') = J_B(01, \eta\eta') + J_B(10, \eta\eta')$$

$$T^y(\eta\eta') = i[J_B(10, \eta\eta') - J_B(01, \eta\eta')]$$

$$T^z(\eta\eta') = J_B(00, \eta\eta') - J_B(11, \eta\eta')$$

By going into the Bell basis the final expression for the fidelity is obtained.

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